BIAXIAL TENSION OF A THICK PLATE MADE OF A STRAIN-HARDENING ELASTOPLASTIC MATERIAL AND CONTAINING A CIRCULAR HOLE

M. A. Artemov

UDC 539.214;539.374

This article gives an approximate solution of a two-dimensional problem on the biaxial tension of a thick plate (plane strain) made of a strain-hardening elastoplastic material and containing a circular hole. The theory of plastic flow with transcendental strain-hardening proposed by A. Yu. Ishlinskii [1] is used. The solution is obtained by the small parameter method developed for elastoplastic problems of the theory of ideal plasticity [2].

L. A. Galinyi [3] examined the biaxial tension of a thick plate made of an ideal elastoplastic material and containing a circular hole. It was shown in [2] that two approximations are sufficient to describe the exact solution. Here we determine two approximations for the same problem with allowance for strain-hardening, and we offer estimates of the effect of strain-hardening on the plastic behavior of the material.

1. In accordance with the theory of translational strain-hardening [1], we can write the loading function for the case of plane strain as follows in polar coordinates r and θ

$$[\sigma_r - \sigma_\theta - c(e_r^p - e_\theta^p)]^2 + 4(\tau_{r\theta} - ce_{r\theta}^p)^2 = 4k^2, \tag{1.1}$$

where σ_r, σ_θ, $\tau_{r\theta}$ are stress components; e_r^p, e_θ^p, $e_{r\theta}^p$ are plastic strain components; c is the strain-hardening parameter; k is the plastic limit.

The total-strain components e_r, e_θ, $e_{r\theta}$ are made up of the elastic and plastic components:

$$(e_r, e_\theta, e_{r\theta}) = (e_r^e, e_\theta^e, e_{r\theta}^e) + (e_r^p, e_\theta^p, e_{r\theta}^p), \tag{1.2}$$

where e_r^e, e_θ^e, $e_{r\theta}^e$ are components of elastic strain.

The elastic strains will be assumed to be incompressible; the plastic strains are incompressible by virtue of the associative law of plastic flow used below. In the case of plane strain, we have the relations

$$e_r^e = -e_\theta^e = \frac{1}{4G}(\sigma_r - \sigma_\theta), \quad e_{r\theta}^e = \frac{1}{2G}\tau_{r\theta}, \tag{1.3}$$

where G is the shear modulus. It follows from the associative flow law that

$$de_r^p + de_\theta^p = 0, \quad \frac{de_r^p - de_\theta^p}{\sigma_r - \sigma_\theta - c(e_r^p - e_\theta^p)} = \frac{de_{r\theta}^p}{\tau_{r\theta} - ce_{r\theta}^p}. \tag{1.4}$$

Integrating the first equation of (1.4), we find that the sum $e_r^p + e_\theta^p$ is independent of the load parameter. If plastic strains are absent at the initial moment of time, then

$$e_r^p + e_\theta^p = 0. \tag{1.5}$$

In accordance with (1.2), (1.3), and (1.5), we have the following condition of incompressibility for the total strains

$$e_r + e_\theta = 0. \tag{1.6}$$

We have the following relations for the total-strain components

$$e_r = \frac{\partial u}{\partial r}, \quad e_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}, \quad e_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \frac{1}{r}\frac{\partial u}{\partial \theta}\right], \tag{1.7}$$

Voronezh. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 158-163, November-December, 1985. Original article submitted August 10, 1984.

0021-8944/85/2606-0908$09.50

where u and v are components of the displacement in the radial and circumferential directions.

First we will examine the elastoplastic equilibrium of a thick-walled axisymmetric tube of radii a and b ($a < b$) subjected to internal and external pressures p_0 and p. It is obvious that in this case $v = e_{r\theta} = \tau_{r\theta} = 0$.

We will refer all quantities having the dimension of length to the radius of the boundary separating the elastic and plastic regions r_s. All quantities having the dimension of stress will be referred to the plastic limit k. We designate

$$\alpha = a/r_s,\ \beta = b/r_s,\ \rho = r/r_s,\ q_0 = p_0/k,$$
$$q = p/k,\ \sigma_\rho = \sigma_r/k,\ \tau_{\rho\theta} = \tau_{r\theta}/k.$$

We keep the former notation for the dimensionless quantities σ_θ, c, G, u, and v. In the elastic zone of the tube ($1 \le \rho \le \beta$) we will have [4]

$$\left.\begin{matrix}\sigma_\rho\\ \sigma_\theta\end{matrix}\right\} = A \mp B\frac{\beta^2}{\rho^2},\quad e_\theta = -e_\rho = \frac{B\beta^2}{2G\rho^2},\quad u = \frac{B\beta^2}{2G\rho}\quad (A, B - \text{const}). \tag{1.8}$$

The boundary conditions:

$$\sigma_{\rho|\rho=\alpha} = -q_0,\ \sigma_{\rho|\rho=\beta} = -q. \tag{1.9}$$

In the plastic zone ($\alpha \le \rho \le 1$), in accordance with (1.1) we write

$$\sigma_\theta - \sigma_\rho - c\left(e_\theta^p - e_\rho^p\right) = 2\varkappa, \tag{1.10}$$

where κ is the sign of the expression on the left side.

We find the following from the condition of continuity of the stress components at $\rho = 1$, the second boundary condition of (1.9), and (1.10)

$$A = -q + B,\ B = \varkappa/\beta^2. \tag{1.11}$$

Compressibility condition (1.6) is valid everywhere: in the elastic and plastic zones; it follows from this that the expressions for the components of the displacements and strains (1.8) are valid in both zones. Then, considering (1.2), (1.3), and (1.8)-(1.11), we obtain the following from the equilibrium equations [3]

$$\sigma_\rho = -q_0 + \frac{G\varkappa}{2G+c}\left[4\ln\frac{\rho}{\alpha} + \frac{c}{G}\left(\frac{1}{\alpha^2} - \frac{1}{\rho^2}\right)\right],$$
$$\sigma_\theta = -q_0 + \frac{G\varkappa}{2G+c}\left[4 + 4\ln\frac{\rho}{\alpha} + \frac{c}{G}\left(\frac{1}{\alpha^2} + \frac{1}{\rho^2}\right)\right].$$

The condition of continuity of the stress components with $\rho = 1$ gives an equation linking the difference $q - q_0$ and the radius r_s:

$$\frac{\varkappa}{\beta^2} = q - q_0 + \frac{\varkappa}{2G+c}\left[2G(1 - 2\ln\alpha) + \frac{c}{\alpha^2}\right]. \tag{1.12}$$

It follows from (1.12) that $\kappa = \operatorname{sign}(q_0 - q)$. When $c = 0$, Eq. (1.12) coincides with the equation obtained for an ideally elastoplastic material [2]. In the case of tension of a plane with a circular hole $\beta \to \infty$, Eq. (1.12) is written as

$$(2G + c)\,|q_0 - q| - 2G(1 - 2\ln\alpha) - \frac{c}{\alpha^2} = 0. \tag{1.13}$$

If $c \ll 1$, then by using (1.13) and representing the equation of the boundary between the elastic and plastic zones in the form $r_s = \sum_{i=0}^{\infty} c^i r_s^{(i)}$, we obtain

$$r_s^{(0)} = a\exp\left(\frac{|q_0 - q| - 1}{2}\right),\quad \rho_s^{(1)} = \frac{1}{4G}\left(1 - \frac{1}{\alpha_0^2} - 2\ln\alpha_0\right), \tag{1.14}$$
$$\rho_s^{(2)} = \frac{\rho_s^{(1)}}{2}\left(\rho_s^{(1)} - \frac{1}{\alpha_0^2 G}\right),\quad \alpha_0 = \frac{a}{r_s^{(0)}},\quad \rho_s^{(i)} = \frac{r_s^{(i)}}{r_s^{(0)}}.$$

The subscript 0 with α will henceforth be omitted. Everywhere below, we take the radius of the boundary between the elastic and plastic zones for an ideal elastoplastic material $r_s^{(0)}$ as the characteristic length scale.

2. We will examine the biaxial tension of an infinite plate with a circular hole of radius a which is subjected at infinity to mutually perpendicular tensile forces p_1 and p_2. Meanwhile, a normal pressure p_0 acts on the contour of the hole. We will seek a solution by the small parameter method [2], assuming that the plastic zone completely envelops the interval contour.

We assume that

$$c = \delta c^*, \ (p_1 - p_2)/2k = \delta p^*,$$

where $\delta << 1$, while c^* and p^* are constants taking values from 0 to 1. At $p^* = 0$ and $c^* = 1$, we have the problem examined above. At $c^* = 0$ and $p^* = 1$, we have the problem examined in [2].

All the stress, strain, and displacement components will be sought in the form of series in powers of δ:

$$(\sigma_{ij}, e_{ij}, u, v) = \sum_{n=0}^{\infty} \delta^n \left(\sigma_{ij}^{(n)}, e_{ij}^{(n)}, u^{(n)}, v^{(n)}\right). \tag{2.1}$$

When $\delta = 0$, the plate is in an axisymmetric state $v^{(0)} = e_{\rho\theta}^{(0)} = \tau_{\rho\theta}^{(0)} = 0$. Inserting (2.1) into (1.1) and equating the terms with identical powers of δ, we obtain

$$\sigma_\theta^{(0)} - \sigma_\rho^{(0)} = 2\varkappa, \quad \sigma_\theta^{(1)} - \sigma_\rho^{(1)} - c^*\left(e_\theta^{(0)p} - e_\rho^{(0)p}\right) = 0,$$
$$\varkappa\left[\sigma_\theta^{(2)} - \sigma_\rho^{(2)} - c^*\left(e_\theta^{(1)p} - e_\rho^{(1)p}\right)\right] + \left(\tau_{\rho\theta}^{(1)}\right)^2 = 0. \tag{2.2}$$

After linearization, Eqs. (1.4) and (1.6) take the form

$$e_\rho^{(n)} + e_\theta^{(n)} = 0, \ n \geqslant 0, \quad e_{\rho\theta}^{(0)} = 0, \ \left(de_\theta^{(0)p} - de_\rho^{(0)p}\right)\tau_{\rho\theta}^{(1)} = 2\varkappa de_{\rho\theta}^{(1)p},$$
$$\left(de_\theta^{(0)p} - de_\rho^{(0)p}\right)\left(\tau_{\rho\theta}^{(2)} - c^* e_{\rho\theta}^{(1)p}\right) + \left(de_\theta^{(1)p} - de_\rho^{(1)p}\right)\tau_{\rho\theta}^{(1)} = 2\varkappa de_{\rho\theta}^{(2)p}. \tag{2.3}$$

We have the following for an infinite plate in the zeroth approximation [2] ($\kappa = 1$)

$$\sigma_\rho^{(0)p} = -q_0 + 2\ln\frac{\rho}{\alpha}, \quad \sigma_\theta^{(0)p} = \sigma_\rho^{(0)p} + 2, \quad \tau_{\rho\theta}^{(0)p} = 0,$$
$$\left.\begin{matrix}\sigma_\rho^{(0)e} \\ \sigma_\theta^{(0)e}\end{matrix}\right\} = q \mp \frac{1}{\rho^2}, \quad \tau_{\rho\theta}^{(0)e} = 0, \text{ where } q = \frac{p_1 + p_2}{2k}. \tag{2.4}$$

Here and below, the stress and displacement components have the superscript e in the elastic zone and p in the plastic zone. Everywhere in the plate

$$u^{(0)} = \frac{1}{2G\rho}, \quad e_\theta^{(0)} = -e_\rho^{(0)} = \frac{1}{2G\rho^2}. \tag{2.5}$$

The boundary between the elastic and plastic zones in the zeroth approximation is determined by the first equation in (1.14).

Let us determine the first approximation. Plastic strains should be found in accordance with the second equation of (2.2). From (1.2), (1.3), and (2.5)

$$e_\rho^{(0)p} = -e_\theta^{(0)p} = \frac{1}{2G}\left(1 - \frac{1}{\rho^2}\right). \tag{2.6}$$

By virtue of linearity, the equilibrium equations retain their form for any approximation:

$$\frac{\partial\sigma_\rho^{(n)}}{\partial\rho} + \frac{\sigma_\rho^{(n)} - \sigma_\theta^{(n)}}{\rho} + \frac{1}{\rho}\frac{\partial\tau_{\rho\theta}^{(n)}}{\partial\theta} = 0,$$
$$\frac{\partial\tau_{\rho\theta}^{(n)}}{\partial\rho} + \frac{1}{\rho}\frac{\partial\sigma_\theta^{(n)}}{\partial\theta} + \frac{2\tau_{\rho\theta}^{(n)}}{\rho} = 0. \tag{2.7}$$

Equations (2.7) can be satisfied by assuming

$$\sigma_\rho^{(n)} = \frac{1}{\rho}\frac{\partial \Phi^{(n)}}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \Phi^{(n)}}{\partial \theta^2}, \tag{2.8}$$
$$\sigma_\theta^{(n)} = \frac{\partial^2 \Phi^{(n)}}{\partial \rho^2}, \quad \tau_{\rho\theta}^{(n)} = -\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial \Phi^{(n)}}{\partial \theta}\right).$$

From (2.2), (2.6), and (2.8) we obtain

$$\frac{\partial^2 \Phi^{(1)}}{\partial \rho^2} - \frac{1}{\rho}\frac{\partial \Phi^{(1)}}{\partial \rho} - \frac{1}{\rho^2}\frac{\partial^2 \Phi^{(1)}}{\partial \theta} = \frac{c^*}{G}\left(\frac{1}{\rho^2} - 1\right). \tag{2.9}$$

The boundary conditions on the inside contour of the plastic zone have the form ($\rho = \alpha$)

$$\sigma_\rho^{(n)p} = \tau_{\rho\theta}^{(n)p} = 0, \quad n \geqslant 1. \tag{2.10}$$

In accordance with (2.9), (2.8), and (2.10), we find

$$\sigma_\rho^{(1)p} = \frac{c^*}{G}\left(\frac{1}{\alpha^2} - \frac{1}{\rho^2} - 2\ln\frac{\rho}{\alpha}\right), \tag{2.11}$$
$$\sigma_\theta^{(1)p} = \sigma_\rho^{(1)p} + \frac{c^*}{G}\left(\frac{1}{\rho^2} - 1\right), \quad \tau_{\rho\theta}^{1(p)} = 0.$$

The boundary conditions at infinity have the form [2]

$$\sigma_\rho^{(1)\infty e} = -p^*\cos 2\theta, \quad \tau_{\rho\theta}^{(1)\infty e} = p^*\sin 2\theta; \tag{2.12}$$
$$\sigma_\rho^{(n)\infty e} = \tau_{\rho\theta}^{(n)\infty e} = 0, \quad n \geqslant 2 \tag{2.13}$$

The compatibility condition for the strain components in the first approximation yields [2]

$$\sigma_\rho^{(1)e} = \sigma_\rho^{(1)p}, \quad \tau_{\rho\theta}^{(1)e} = \tau_{\rho\theta}^{(1)p} \quad \text{at} \quad \rho = 1. \tag{2.14}$$

Using boundary conditions (2.12) and (2.14), we write the expressions for the stress and displacement components in the elastic zone:

$$\sigma_\rho^{(1)e} = \frac{a_0}{\rho^2} - p^*\left(1 - \frac{4}{\rho^2} + \frac{3}{\rho^4}\right)\cos 2\theta, \tag{2.15}$$
$$\sigma_\theta^{(1)e} = -\frac{a_0}{\rho^2} + p^*\left(1 + \frac{3}{\rho^4}\right)\cos 2\theta, \quad \tau_{\rho\theta}^{(1)e} = p^*\left(1 + \frac{2}{\rho^2} - \frac{3}{\rho^4}\right)\sin 2\theta,$$
$$u^{(1)e} = -\frac{a_0}{2G\rho} - \frac{p^*}{2G}\left(\rho + \frac{2}{\rho} - \frac{1}{\rho^3}\right)\cos 2\theta,$$
$$v^{(1)e} = \frac{p^*}{2G}\left(\rho + \frac{1}{\rho^3}\right)\sin 2\theta, \quad \text{где} \quad a_0 = \frac{c^*}{2G}\left(\frac{1}{\alpha^2} - 1 + 2\ln\alpha\right).$$

Using the compatibility conditions $\left[\sigma_\theta^{(1)} + \frac{d\sigma_\theta^{(0)}}{d\rho}\rho_s^{(1)}\right] = 0$ with $\rho = 1$ and following [2], we obtain

$$\rho_s^{(1)} = -\frac{a_0}{2} + p^*\cos 2\theta. \tag{2.16}$$

From the displacement compatibility condition [2], which reduces to the form

$$u^{(1)e} = u^{(1)p}, \quad v^{(1)e} = v^{(1)p} \quad \text{at} \quad \rho = 1,$$

and from Eqs. (2.15) we obtain the boundary conditions for the displacements in the plastic zone

$$u^{(1)p} = \frac{a_0}{2G} - \frac{p^*}{G}\cos 2\theta, \quad v^{(1)p} = \frac{p^*}{G}\sin 2\theta. \tag{2.17}$$

From (2.3) and (2.11) we have

$$e_{\rho}^{(1)}+e_{\theta}^{(1)}=0,\quad de_{\rho\theta}^{(1)p}=0. \tag{2.18}$$

Following [2], we note that since $de_{\rho\theta}^{(1)p}=d\lambda\frac{\partial e_{\rho\theta}^{(1)p}}{\partial\lambda}$ (λ is the load parameter), then in accordance with (2.17), $e_{\rho\theta}^{(1)p}$ is independent of the change in load. At the initial moment the plastic strains are equal to zero, which means

$$e_{\rho\theta}^{(1)p}=e_{\rho\theta}^{(1)}-\frac{1}{2G}\tau_{\rho\theta}^{(1)p}=e_{\rho\theta}^{(1)}=0. \tag{2.19}$$

With allowance for (1.7), Eqs. (2.18) and (2.19) take the form

$$\frac{\partial u^{(1)p}}{\partial\rho}+\frac{u^{(1)p}}{\rho}+\frac{1}{\rho}\frac{\partial v^{(1)p}}{\partial\theta}=0,\quad \frac{\partial v^{(1)p}}{\partial\rho}-\frac{v^{(1)p}}{\rho}+\frac{1}{\rho}\frac{\partial u^{(1)p}}{\partial\theta}=0. \tag{2.20}$$

Assuming $u^{(1)p}=-\frac{1}{\rho}\frac{\partial\Psi^{(1)}}{\partial\theta}$, $v^{(1)p}=\frac{\partial\Psi^{(1)}}{\partial\rho}$, we obtain the following from the second equation of (2.20)

$$\frac{\partial^2\Psi^{(1)}}{\partial\rho^2}-\frac{1}{\rho}\frac{\partial\Psi^{(1)}}{\partial\rho}-\frac{1}{\rho^2}\frac{\partial^2\Psi^{(1)}}{\partial\theta^2}=0. \tag{2.21}$$

Using the solution of Eq. (2.21) presented in [2] and boundary conditions (2.17), we have

$$u^{(1)p}=-\frac{a_0}{2G\rho}-\frac{2p^*}{\sqrt{3}\,G}\cos\left(t-\frac{\pi}{6}\right)\cos 2\theta,\quad v^{(1)p}=\frac{2p^*}{\sqrt{3}\,G}\cos\left(t+\frac{\pi}{6}\right)\sin 2\theta, \tag{2.22}$$

$$e_{\rho}^{(1)}=-e_{\theta}^{(1)}=\frac{a_0}{2G\rho^2}+\frac{2p^*}{G\rho}\sin\left(t-\frac{\pi}{6}\right)\cos 2\theta,\quad e_{\rho\theta}^{(1)}=0,$$

where $t=\sqrt{3}\ln\rho$.

Thus, the first approximation has been found completely. In contrast to the solution for an ideal elastoplastic material [2], the expression for the boundary between the elastic and plastic zones (2.16) contains a constant $- a_0/2 -$ which characterizes the "slowing" of propagation of the plastic zone due to strain-hardening.

3. Let us proceed to the determining of the second approximation. From (2.2) and (2.11) we find

$$\sigma_{\theta}^{(2)p}-\sigma_{\rho}^{(2)p}=c^*\left(e_{\theta}^{(1)p}-e_{\rho}^{(1)p}\right). \tag{3.1}$$

In accordance with (1.2), (1.3), (2.11), and (2.22), we will have

$$e_{\theta}^{(1)p}-e_{\rho}^{(1)p}=\frac{c^*}{2G^2}\left(1-\frac{1}{\rho^2}\right)-\frac{a_0}{G\rho^2}-\frac{4p^*}{G\rho}\sin\left(t-\frac{\pi}{6}\right)\cos 2\theta. \tag{3.2}$$

After insertion of (2.8) and (3.2) into (3.1), we obtain an equation analogous to (2.9). By solving this equation using Eqs. (2.8) and by taking boundary conditions (2.10) into account, we find

$$\sigma_{\rho}^{(2)p}=\frac{c^*}{2G}\left[\frac{c^*}{2G}\left(2\ln\frac{\rho}{\alpha}+\frac{1}{\rho^2}-\frac{1}{\alpha^2}\right)+a_0\left(\frac{1}{\rho^2}-\frac{1}{\alpha^2}\right)\right]+\frac{4c^*p^*}{\sqrt{3}G\rho}\left[\sin t-t_1\sin\left(t+\frac{\pi}{6}\right)-\sin t_0\cos t_1\right]\cos 2\theta, \tag{3.3}$$

$$\sigma_{\theta}^{(2)p}=\sigma_{\rho}^{(2)p}+\frac{c^*}{G}\left[\frac{c^*}{2G}\left(1-\frac{1}{\rho^2}\right)-\frac{a_0}{\rho^2}-\frac{4p^*}{\rho}\sin\left(t-\frac{\pi}{6}\right)\cos 2\theta\right],$$

$$\tau_{\rho\theta}^{(2)p}=\frac{4c^*p^*}{\sqrt{3}G\rho}\left[\cos\left(t-\frac{\pi}{6}\right)-\cos\left(t_0-\frac{\pi}{6}\right)\cos t_1-t_1\sin\left(t-\frac{\pi}{6}\right)\right]\sin 2\theta,$$

where

$$t_0=\sqrt{3}\ln\alpha;\ t_1=\sqrt{3}\ln\frac{\rho}{\alpha}.$$

The stress compatibility conditions in the second approximation have the form [2]

$$\left[\sigma_\rho^{(2)} + \frac{d\sigma_\rho^{(1)}}{d\rho}\rho_s^{(1)} + \frac{d^2\sigma_\rho^{(0)}}{d\rho^2}\frac{\rho_s^{(1)2}}{2} + \frac{d\sigma_\rho^{(0)}}{d\rho}\rho_s^{(2)}\right] = 0. \tag{3.4}$$

The compatibility conditions for σ_θ and $\tau_{\rho\theta}$ are similar. Considering (3.3), (2.4), (2.11), (2.15), and (2.16), we obtain the following from (3.4) (ρ = 1)

$$\sigma_\rho^{(2)e} = b_0 + c_1\cos 2\theta - p^{*2}\cos 4\theta,\ \tau_{\rho\theta}^{(2)e} = c_2\sin 2\theta - 4p^{*2}\sin 4\theta, \tag{3.5}$$

where

$$b_0 = -p^{*2} - \frac{a_\theta}{2}\left(a_0 + \frac{c^*}{G\alpha^2}\right);\ c_1 = 2p^*\left[a_0 + \frac{c^*}{G}\left(\ln\alpha - \frac{1}{\sqrt{3}}\sin 2t\right)\right];$$

$$c_2 = 2p^*\left[2a_0 - \frac{c^*}{G}\left(\ln\alpha - \sin^2 t_0 + \frac{1}{2\sqrt{3}}\sin 2t_0\right)\right].$$

In accordance with boundary conditions (3.5) and (2.13), we have the following for the second approximation in the elastic zone

$$\sigma_\rho^{(2)e} = \frac{b_0}{\rho^2} + \left(\frac{2M}{\rho^2} - \frac{N}{\rho^4}\right)\cos 2\theta + p^{*2}\left(\frac{9}{\rho^4} - \frac{10}{\rho^6}\right)\cos 4\theta, \tag{3.6}$$

$$\sigma_\theta^{(2)e} = -\frac{b_0}{\rho^2} + \frac{N}{\rho^4}\cos 2\theta - p^{*2}\left(\frac{3}{\rho^4} - \frac{10}{\rho^6}\right)\cos 4\theta,$$

$$\tau_{\rho\theta}^{(2)e} - \left(\frac{M}{\rho^2} - \frac{N}{\rho^4}\right)\sin 2\theta + 2p^{*2}\left(\frac{3}{\rho^4} - \frac{5}{\rho^6}\right)\sin 4\theta,$$

where $M = c_1 - c_2$; $N = c_1 - 2c_2$. We find the following from the compatibility condition for σ_θ with ρ = 1 in the second approximation

$$\rho_s^{(2)} = \frac{a_0}{8} + \frac{c^* a_0}{4G\alpha^2} - p^*\left[\frac{a_0}{2} - \frac{c^*}{G}\left(\ln\alpha - \sin^2 t_0 + \frac{1}{2\sqrt{3}}\sin 2t_0\right)\right]\cos 2\theta - \frac{3}{4}p^{*2}(1 - \cos 4\theta). \tag{3.7}$$

We will make several observations. Equations (2.11), (2.15), (2.16), (2.22), (3.3), (3.6), and (3.7) make it possible to evaluate the effect of strain-hardening. At $c^* = 0$, we have the solution presented in [2]. At $p^* = 0$, Eqs. (2.16) and (3.7) coincide with (1.14).

We should point out that Eq. (2.3) includes differentials of the strain components. In the given case, $de_{ij}^p \approx d\lambda \partial e_{ij}^p/\partial\lambda$. In problems similar to those examined above, it is convenient to take ρ as the load parameter. Integration over the plastic strains should be done from zero to the running value of the plastic strains, while integration over ρ should be done from unity to the running value of ρ.

LITERATURE CITED

1. A. Yu. Ishlinskii, "General theory of elasticity with linear strain-hardening," Ukr. Mat. Zh., 6, No. 3 (1954).
2. D. D. Ivlev and L. V. Ershov, Perturbation Method in the Theory of Elastoplastic Bodies [in Russian], Nauka, Moscow (1978).
3. V. V. Sokolovskii, Theory of Plasticity [in Russian], Vysshaya Shkola, Moscow (1969).
4. M. M. Filonenko-Borodich, Theory of Elasticity [in Russian], Gosizdat, Moscow (1959).